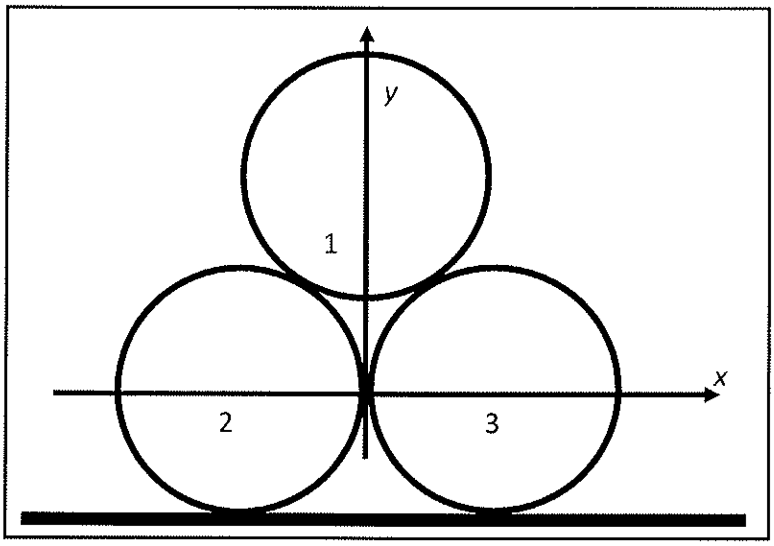
1. Spheres in Contact

A system of three identical rigid spheres of radius R, are put in contact on a horizontal table, in gravitational field of intensity g, as shown in figure. At the initial moment the system is released. Considering a frictionless motion obtain:



a) Velocity of the spheres at the detachment time as function of y coordinate (see the figure);
b) $v_3^2/(gR)$ as function of y/R, where v_3 is the speed of sphere 3 (see the figure); justify the correctness of solutions for the speed of the spheres;
c) The maximum possible height of sphere 1 after it elastically collides the horizontal table;

One considers the real solution of the equation $x^3+12x-8\sqrt{3}=0$ is known, namely, $x_0 \approx 1.056$.

Prof. Tiberius O. Cheche-University of Bucharest

2. The apparent air temperature in a windy winter day

In a calm winter day (with no wind) thermometers indicate the average air temperature of -5 ^{0}C. However, human subjects (whose average skin temperature is 37 ^{0}C) perceive approximately 5 ^{0}C in these conditions.

Assume in that same day a strong turbulence occurs, a strong wind, with no change in the actual air temperature. Estimate the new perception of the human body in this modified environment. In other words, find out which value should have the outside temperature of a calm atmosphere in order to make people feel the same cold than the -5 ^{0}C air of that windy day.

Prof. Valeriu Filip - University of Bucharest

3. Huygens cycloidal pendulum

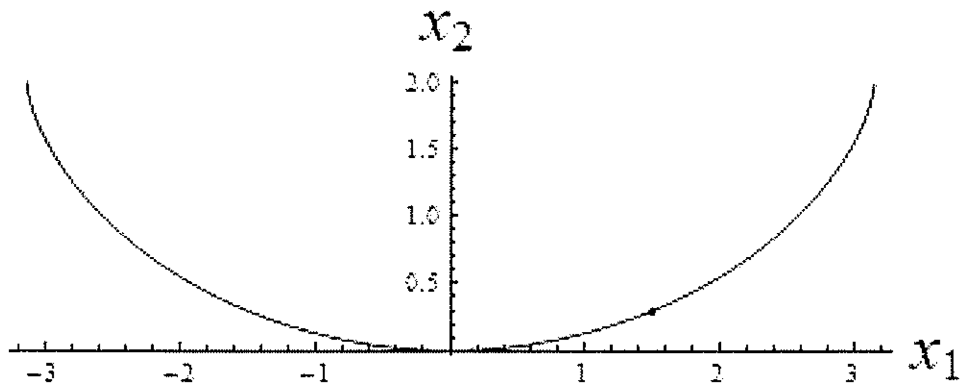
The Dutch mathematician and physicist Christiaan Huygens invented the cycloidal pendulum clock in December 1656; he was 27 years old. The Rijksmuseum voor de Geschiedenis der Natuurwetenschapen in Leiden has the clock made by Huygens in 1657, which has cycloidal chops, this being the oldest pendulum clock.

The *cycloidal pendulum* is described by a particle that moves without friction on a vertical cycloid. The parametric equations of the cycloid are (see the figure bellow):

$x_1 = a\,(\alpha + \sin\alpha\,);$

$x_2 = a\,(1 - \cos\alpha\,),$

with $\alpha \in [-\pi, \pi]$. We consider that the particle starts moving from rest from its highest point from the left, i.e. $x_1(t = 0) = -a\,\pi$ and $x_2(t = 0) = 2\,a$.



1. Find the expression of the Lagrange function.

2. Determine the law of motion.

3. Find the period of oscillation.

Prof. Iulia Ghiu - University of Bucharest

4. Piezoelectric voltage from super-currents

A cylinder shaped piezo stacked transducer is made of N circular (radius R) well insulated superconducting turns, embedded in a piezoelectric ceramic with no magnetic properties (FIG.1). Without super-current flowing in turns, they are separated by some small distance d. The ceramic material obeys Hooke's law and its stiffness is k. Under stretching or compression of ceramic with $N\Delta d$ along the cylinder axis, a voltage drop $U = \gamma N\Delta d$ develops between the top and the bottom of the stack, where γ is a constant specific to the ceramic material. Such an effect is known as *piezoelectric effect*. Note that *a superconducting material exhibits exactly zero electric resistance*. Superconductivity was discovered by Dutch physicist H. Kamerlingh Onnes on April 8, 1911 in Leiden. To solve the problem, you only need the basic knowledge of mechanics, electricity and magnetism.

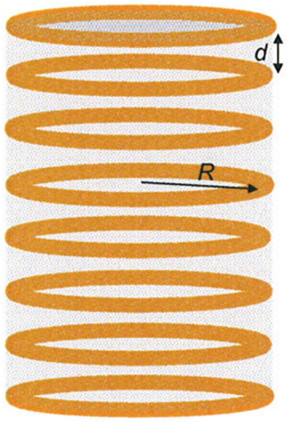


FIG.1: A piezo stacked transducer made of N circular (radius R) superconducting turns, embedded in a piezoelectric ceramic.

1. Show that when super-currents I flow in turns, a voltage drop develops across the stacked transducer.
2. Use simple considerations to find if the voltage exhibits even or odd dependence on I.
3. With some super-current flowing in turns, analize what will happen to the magnetic flux, when trying to compress or strech the piezo stacked transducer.
4. Find the piezoelectric voltage dependence on super-current.

Prof. Petrica Cristea - University of Bucharest

5. SUM RULES

Consider a quantum system defined by the Hamiltonian H having the eigenenergies E_n and eigenstates $|E_n\rangle$ which form a complete orthonormal basis, $H|E_n\rangle = E_n|E_n\rangle$.

The excitation strength function associated with a Hermitian excitation operator F is defined as:

$$S(E) = \sum_{n\neq 0} |\langle E_n|\, F\, |E_0\rangle|^2\ \delta(E - (E_n - E_0))$$

Show that the moments of the strength function $m_k = \int_0^\infty E^k S(E)dE$ can be written as the expectation value on the ground state of an operator function depending on H and F.

Indicate the physical meaning of m_1 and prove that it can be expressed as $m_1 = \frac{1}{2}\langle E_0|[F,[H,F]]|E_0\rangle$, which represents the energy weighted sum rule.

Suppose that our system consists of A nucleons interacting through momentum independent forces. Consider the application upon the system of a perturbation of the form $F = \sum_{i=1}^{A} f(\vec{Q}_i)$, where $\vec{Q}_i$ is the position operator associated to the nucleon i. Prove that in this case

$$m_1 = \frac{\hbar^2}{2m}\left\langle E_0\right| \sum_{i=1}^{A}\left(\nabla f(\vec{Q}_i)\right)^2 \left|E_0\right\rangle.$$

Provide a physical interpretation of the right hand side of this equation. Having in mind the physical meaning of m_1 you provided earlier, indicate which fundamental law of nature is encoded in the energy weighted sum rule.

The dipolar response of atomic nuclei will be explored at the ELI-NP facility in Magurele with gamma rays obtained from the process of Compton backscattering. In this case

$$F = \frac{NZ}{A}\left(\frac{1}{Z}\sum_{p=1}^{Z} Q_{z,p} - \frac{1}{N}\sum_{n=1}^{N} Q_{z,n}\right)$$

where $Q_{z,p}$ is the z component of the position operator for protons, and similarly for $Q_{z,n}$ in the case of neutrons. Here $A = N + Z$, with N the total number of neutrons and Z the total number of protons. Then the dipolar cross section for the absorption of a photon with energy E can be expressed as:

$$\sigma(E) = \frac{4\pi^2 e^2}{\hbar c} E\, S(E)$$

If we assume, as above, that the nucleons interact through momentum independent forces determine the total cross section $\sigma_D = \int_0^\infty \sigma(E)\, dE$ for this process.

Prof. Virgil Baran - University of Bucharest

6. DYNAMICAL SYMMETRIES

Consider the motion of a particle with mass m in a central field determined by the Hamiltonian:

$$H = \frac{\vec{P}^2}{2m} - \frac{\alpha}{Q_r}$$

where $\vec{P}$ is the momentum operator and $Q_r = \sqrt{\vec{Q}^2}$. For this potential, within classical dynamics, an additional conserved quantity is the Runge-Lenz vector. Motivate why in quantum mechanics the appropriate definition for the Runge-Lenz operator is $\vec{M} = \frac{1}{2m}\left(\vec{P} \times \vec{L} - \vec{L} \times \vec{P}\right) - \alpha\frac{\vec{Q}}{Q_r}$, and not $\vec{M} = \frac{1}{m}\vec{P} \times \vec{L} - \alpha\frac{\vec{Q}}{Q_r}$ as in the classical expression.

i) Prove that $\vec{M}$ is a constant of motion also in the quantum case.

ii) Show that $\vec{M}$ behaves like a vector operator under rotations and that $\vec{M} \cdot \vec{L} = 0$.

In addition to the $SO(3)$ rotational symmetry due to the fact that the Coulomb potential is radial, the conservation of $\vec{M}$ enlarges the symmetry group of the Hydrogen atom to $SO(4)$. However, the additional symmetry does not have a geometrical correspondence as in the case of rotations, and hence is named 'dynamical'.

Prof. Virgil Baran - University of Bucharest

7. Neutron star

It is done the semi-empirical mass formula:

$$M(A,Z) = ZM_H + NM_n - a_V A + a_S A^{\frac{2}{3}} + a_C \frac{Z}{A^{\frac{1}{3}}} + a_A \frac{(A-2Z)^2}{A} \pm a_P \frac{1}{A^{\frac{1}{2}}}$$

The following are required:

(a) to add of that formula a term which give the potential energy due to gravitational attraction and to establish the form of this potential energy;

(b) to apply this new form of formula for a neutron star and to find out the smallest radius of neutron star, under the following hypothesis:

(i) the nuclear density is the same everywhere;

(ii) the neutron star does not contain other constituents.

It is given: a_V = *15.56 MeV*; a_A = *23.29 MeV*; a_S = *18.33MeV*; a_C = *0.691 MeV*; $g = 6.673x10^{-11} m^3 kg^{-1} s^{-2}$

Prof. Alexandru Jipa - University of Bucharest

8. Elementary particles

The resonance Δ^{++} (their mass is 1232 MeV/c^2) has a full with of $\Gamma = 120\ MeV$.

a) How far on average would such a particle of kinetic energy 200 GeV travel before decaying?

b) This resonance decay following the process: $\Delta^{++} \rightarrow X + \pi^{+}$. Determine the following properties of X: i) electric charge; ii) baryon number, lepton number, isospin, strangeness, it is boson or fermion; iii) lower limit on its mass in MeV/c^2; iv) identify this particle.

c) Show the relationship between the pion energy in the laboratory frame (E_{π}) and its angle relative to the Δ^{++} beam (θ), for small angles.

d) In the frame of the quark model draw the diagram of the decay process.

($\hbar c = 197\ MeV \cdot fm;\ \ c = 3 \cdot \frac{10^8 m}{s}, m_{\pi^+}\ = 140 MeV/c^2, m_p = 940 MeV/c^2$)

Prof. Ionel Lazanu - University of Bucharest

9. Rapidity and reference systems

Are given a reference frame S and another reference frame S'. If the movement is performed over a single axis, with the relative speed β, demonstrate that there is the following relationship between rapidities in the two reference systems: $y' = y - y_\beta$, where $y_\beta = \frac{1}{2}\ln\frac{1+\beta}{1-\beta}$.

Prof. Alexandru Jipa - University of Bucharest

10. Langmuir law

A container which contains an ideal gas at the temperature T and pressure P is considered; on a portion of the surface of the container there are N_0 adsorption centers, the number of these adsorption centers being very small comparing with the number total of particles; the adsorbed particles remain located, without translations, and for extraction of a particle from an adsorption center into the gaseous phase it is necessary the energy ε. We consider that in the gaseous phase the particles behave as material points with the mass M (i.e. they have only translation degrees of freedom), and in the adsorbed phase the particles behave as harmonic isotropic oscillator with the frequency ω.

In condition of equilibrium, when a small part of the particles are adsorbed, and the rest of them are in the gaseous phase, we define the *covering ratio* with the relation

$$\theta(T,P) \equiv \frac{\langle N_a \rangle}{N_0} ,$$

where N_0 is number of adsorption centers and $\langle N_a \rangle$ is average number of the adsorbed particles.

1. If we consider that the gaseous system is a particles reservoir for the adsorbed system (i.e. the system of the adsorbed particles is in grand-canonical conditions) we ask to determine:
   - a) the chemical potential of the gaseous phase, considered in canonical conditions, but expressed as function of the temperature and the pressure $\mu(T,P)$;
   - b) the grand-canonical average number of adsorbed particles $\langle N_a \rangle (T; N_0)$, in function of the chemical potential imposed by the gaseous phase;
   - c) the covering ratio $\theta(T,P)$, obtained from the previous results.
2. If we consider the canonical formalism, where the two subsystems have the numbers of particles N_g and N_a we ask to obtain the covering ratio by passing the following stages:
   - a) the derivation of the canonical chemical potential of the gaseous phase $\mu_g(T,V,N_g)$ and $\mu_g(T,P)$;
   - b) the derivation of the canonical chemical potential of the adsorbed system $\mu_a(T,N_a;N_0)$;
   - c) using the condition of chemical equilibrium between the two subsystems $\mu_g = \mu_a$, it results the covering ratio $\theta(T,P)$;
   - d) we ask to show that the two methods (the grand-canonical and the canonical ones) lead to identical results.

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